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ABSTRACT

Throughout the history of mathematics education, many studies have attempted to determine what constitutes "good" mathematics teaching and how such teaching can be put into practice. Missing in the literature, however, are the voices of secondary mathematics teachers and how they define good mathematics teaching. This study contributes to the discussion of characterizing good mathematics teaching by providing the perspectives of secondary mathematics teachers and how those perspectives compare with research and policy in mathematics education. (Contains 24 references.) (MM)

What Constitutes Good Mathematics Teaching: Teachers' perspectives

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The notion of good mathematics teaching is an illusive concept that, nevertheless, has permeated the literature for decades, perhaps centuries. Often the construct is ill-defined and subject to interpretation. It is most often the case that educational scholars emphasize the broader, more process-oriented goals of education. For example, Dewey (1916) defines education as "the reconstruction or reorganization of experiences which add to the meaning of experience, and which increases ability to direct the course of subsequent experiences" (p. 76). Dewey is adamant that education is not just about the learning of the three Rs but should be conceived from a much broader perspective, one in which citizens become literate and capable of directing their own lives through informed and reasoned choices. Such a broad perspective strongly suggests that Dewey would define good teaching as that which enables students to realize these broader educational goals.

Brownell (1956) maintained that a balance between meaning and skill in school mathematics ought not to be held hostage to repetitive practice, a position that promotes a certain pluralism in the teaching of mathematics. It follows that good teaching would involve striking a balance between developing skills and an understanding of the encompassing mathematics. Polya (1965) argued that the primary aim of mathematics teaching is to teach people to think. He wrote, " 'Teaching to think' means that the mathematics teacher should not merely impart information, but should try also to develop the ability of the students to use the information imparted" (p. 100). Polya listed the following ten commandments for teachers.

1. Be interested in subject
2. Know your subject
3. Know about ways of learning (Best to discover it yourself)
4. Read faces of students
5. Give students "know how", attitudes of mind, habit of methodical work
6. Let students learn guessing
7. Let students learn proving

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8. Look at features of problems that suggest solution methods.
9. Let students guess before you tell them.
10. Suggest it, do not force it down their throats. (p. 116)

Polya's commandments emphasize a process-oriented teaching style consistent with Dewey's notion of education and Brownell's call for balance in teaching. We can see at least three emphases: the content (commandments 1 and 2), an emphasis on students' thinking (commandments 3, 4, and 5), and a certain pedagogy or teaching style (commandments 6-10).

Clark and Fehr (1953) considered the teacher a "builder of minds that will solve problems" (p. 348). These authors thought of the teacher as a master technician, one who attends to recent developments in the field in an effort to engender student learning, that is, student thinking. Davis and Hersh (1981) emphasized a certain pluralism in the teaching of mathematics in which students see mathematics as more than a formalistic science. They spoke explicitly against authoritarian presentations. To them, the ideal teacher is one who invites students to "Come, let us reason together," rather than one who uses "proof by coercion" (P. 282).

It seems clear that through the decades, scholars envision teaching as an activity that promotes thinking and problem solving rather than the accumulation of information. This perspective is consistent with standards developed by the National Council of Teachers of Mathematics (NCTM). In the original set of Standards, it is stated that "Finally, our visions sees teachers encouraging students, probing for ideas, and carefully judging the maturity of a student's thoughts and expressions" (NCTM, 1989, p. 10). The Professional Standards for Teaching Mathematics (NCTM, 1991) were based on the following four assumptions:

1. The goal of teaching mathematics is to help all students develop mathematical power;
2. What students learn is fundamentally connected with how they learn it (emphases in original);
3. All students can learn to think mathematically; and
4. Teaching is a complex practice and hence not reducible to recipes or prescriptions (p. 22).

In the Principles and Standards for School Mathematics (2000) the Teaching Principle is stated as follows:

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well. (p. 16)

The Teaching Principle has a decidedly constructivist orientation in that effective teaching is defined in terms of the teacher basing instruction on students' thinking. A condition for effective teaching is that "teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (p.17).

The various statements about teaching in the NCTM Standards clearly embrace the notion of pluralism in teaching and of the importance of teachers connecting with students' understanding of mathematics. From this perspective we can see considerable convergence between the NCTM's definition of effective or good teaching and those "definitions" emanating from scholars such as Dewey, Brownell, and Polya who consistently emphasized teaching as a process that promotes analysis, thinking, and problem solving.

Notion of Good Teaching as Reflected in Research

Although research seldom addresses the issue of good teaching directly, the notion of effective teaching is implicit, particularly when the dominant methodology is deterministic. Since the notion of what constitutes good teaching seems to vary according to the type of research conducted, we divide this section into two subsections that distinguish findings from deterministic studies from interpretivist studies.

Good Teaching and Deterministic Studies

Most of the deterministic studies in education and in mathematics education occurred during the 1960s and 1970s under the guise of either process-product or experimental studies. Rosenshine and Furst's (1971) review of educational research revealed 11 variables from process-product studies that were associated with effective teaching: clarity, variability, enthusiasm, task-oriented or business like behaviors, opportunity to learn, the use of student ideas and general indirectness, criticism, the use of structuring comments, types of questions, probing, and the level of difficulty of instruction. The first five variables were found to be more strongly correlated with student learning than the latter six. In terms of mathematics education, the constructs clarity and variability attracted the most attention. Although these two constructs were high-inference in nature, the view seemed to be that if teaching could be judged clear and flexible that student learning could be improved. That is, these constructs were part of what constituted "good teaching."

Good, Grouws, and Ebmeier's (1983) extensive study of mathematics teaching led to the development of a model that the authors felt represented effective mathematics teaching. Their

model was based on time allocation: review and practice on mental computation (8 minutes), developmental portion of the lesson (20 minutes), seatwork (15 minutes) and assignment of homework (2 minutes). Although the model was offered as a heuristic for teachers, it was sometimes implemented in an algorithmic way, which led the authors to question why teachers differed in their interpretation and implementation of the model.

McKinney's (1986) review of research on expert-novice teachers examined behaviors of these two kinds of teachers. In general, expert teachers were more efficient in organizing and conducting lessons, had a better knowledge of the content than did novice teachers, had clearer lessons, and were more adept at explaining why, how, and when mathematical concepts are used. McKinney quantified differences between expert and novice teachers in the following way:

1. Novice teachers took as long as 15 minutes to correct assigned homework where as expert teachers accomplished this in 2 or 3 minutes.
2. Most expert teachers covered at least 40 problems a day through games, drills, or written work whereas novice teachers covered only 6 or 7 problems.

There is much left unsaid here about the quality of the coverage and the attitude conveyed about mathematics. Our point is that McKinney's study points us in the direction of specific behaviors that are quantifiable and are associated with what was considered to be better teaching. Similarly, Leinhardt (1988) studied expert and novice teachers and found that expert teachers could weave what the author called segments and routines of lessons together in a more coherent and flexible way than could novices and could draw upon richer representations of concepts than could novice teachers. Again, the focus of the research was on highly quantifiable aspects of teaching as a means of characterizing effective mathematics teaching.

Research based on deterministic methodologies was often criticized in terms of the kind of limited mathematics that defined student achievement and thus teacher effectiveness. Further, it was generally the case that the research was grounded in a more teacher-centered orientation to teaching and thus focused on ways of making direct instruction more effective. Although direct instruction is not necessarily contradictory to a student-centered classroom, one has the feeling that effective teaching was defined primarily in terms of what teachers could do as an activity in itself rather than as an activity based on student thinking. In this sense, there is a certain disparity between what this kind of research defined as effective or good teaching and how the NCTM Standards defined good teaching.

Good Teaching and Interpretative Studies

As one might expect, interpretative studies, especially those involving case studies, emphasize teachers' beliefs and actions as seen in the context of classrooms. Here, good teaching must be inferred from what the researcher is studying which is usually not focused on specific behaviors. We will examine several studies that fit this type of methodology as we consider how these researchers defined, implicitly, the notion of good teaching.

Wilson and Goldenberg's (1998) study of Mr. Burt, a middle school mathematics teacher who was interested in reforming his teaching, represents a case in point. The researchers worked with Mr. Burt in an effort to help him reform his teaching and make it more conceptually oriented and aligned with students' thinking. They concluded that, "Mr. Burt's approach generally portrayed mathematics as a rigid subject to be mastered and correctly applied, rather than as a way of thinking or as a subject to be explored" (p. 287). Although the authors do not address the notion of good teaching directly, it seems reasonable to assume that, to them, good teaching involves a student-centered instructional style in which mathematics is treated conceptually. Even though Mr. Burt's teaching became more conceptually oriented, his primary mode of instruction remained teacher-centered thus suggesting that Mr. Burt's notion of good teaching differed from that of the authors.

The notion of good teaching as that in which teachers reflect on their own understanding as well as on their students' understanding is illustrated in the work of Schifter (1998) who concluded that "teachers must come not only to expect, but to seek, situations in their own teaching in which they can view the mathematics in new ways, especially through the perspectives that their students bring to the work" (p. 84). Schifter's approach provided a context in which teachers could experience mathematics not as a formalized subject but as a subject that resulted from human invention. In some sense, Schifter was defining, implicitly, good mathematics teaching as that kind of teaching in which teachers reflect on what students know and consequently see mathematics in terms of the students' constructions of mathematical ideas.

Gellert's (2000) study of 42 elementary pre-service teachers revealed that many of the teachers believed that their role as teachers of mathematics was to protect students from their own frightening mathematical experiences as students. In order to achieve this goal, the teachers introduced child-centered media for instruction but, concomitantly, reduced the emphasis on mathematical content. By minimizing the content, the teachers felt that they were protecting

children from the abstract terrain of mathematics. From the teachers' perspective, it seems clear that good teaching consisted of providing children an anxiety free environment, a perspective that outweighed the importance of children learning mathematics. The researcher took the position that good teaching consisted of allowing children to interact with significant mathematical ideas that would not be "hidden" behind a media/activities approach.

In a study of preservice secondary teachers, Eggleton (1995) and later Chauvot (2000) provided in-depth analyses of preservice teachers' interpretations of their learning experiences in a reform-oriented teacher education program. In particular, Eggleton's Ken entered the program with the belief that mathematics was a set of rules and guidelines that he (Ken) collectively referred to as "number crunching." Ken saw mathematics as certain and made up of absolute truths. His view of teaching was essentially that of telling; learning mathematics consisted primarily of practice, discipline, and memorization. Once Ken's views were challenged, he underwent a re-examination of his beliefs that led him to view mathematics from a quasi-empiricist philosophy. Nevertheless, Ken's teaching remained traditional and teacher centered throughout his student teaching. Similarly, Chauvot found that the preservice teachers embraced a reform-oriented view of teaching, as was espoused throughout their teacher education program, but the teachers' more dominant beliefs remained traditional. For example, Liz, one of the three teachers Chauvot studied, held firmly to the view that "Good mathematics teaching entailed a chain of events in which clear, flexible instruction leads to student success, which leads to student confidence, which leads to further study, appreciation, and a love for mathematics" (p. 99). Another teacher, Stacey, who was adamant that mathematics was more than memorization and subscribed to the importance of meaning, was critical of teachers who emphasized memorization over understanding. Her perspective was, generally speaking, realized in her teaching as she strove to teach mathematics from a pluralistic perspective. The teacher educator's continual emphasis on process-oriented teaching styles had differential effects on the participant teachers. Some, such as Ken and Liz, held to a more traditional view of teaching mathematics, whereas Stacey provided evidence that her notion of good teaching remained consistent with that espoused by her teachers.

Summary

In an effort to quantify effective or good teaching, research grounded in deterministic methodologies strove to identify teaching behaviors that were associated with student learning.

It could be assumed that the appropriate manifestation of those behaviors could provide one definition of good teaching. The notion of good teaching based on interpretivist research is more implicit and must be inferred from the researcher's apparent orientation and from his/her interpretation of the ways teachers taught mathematics. Although current teacher education programs typically promote a student-centered and process-oriented style of teaching, a position consistent with the NCTM Standards, the evidence suggests that teachers who emerge from those programs often do not engage in that kind of teaching. As observed by Skott (2001), teachers' beliefs about mathematics are often buffeted by other more centrally held beliefs and by circumstances particular to schools and society. The question then arises about the compatibility of how teachers define good teaching versus how teacher educators define good teaching. In essence, this study is about examining that compatibility.

Methodology

Participants

Nine secondary mathematics teachers who participated in a three-year Eisenhower-funded project (1997 – 2000) at the University of Georgia entitled PRIME (Partnerships and Reform in Mathematics Education) served as informants for this study. The purpose of Project PRIME was to provide the best field-experience for preservice teachers and mentors by:

- enhancing professional relationships between university faculty and staff, mentor teachers, and preservice teachers;
- providing mentor teachers opportunities to dialogue with one another and university faculty and staff; and
- providing a knowledge base to encourage an open discussion regarding reform mathematics and a consistent experience for preservice teachers between campus and field.

The structure of Project PRIME consisted of requiring participating mentor teachers to not only commit to the traditional Spring Semester student teaching field-experience, but also to a four week Fall Semester field-experience (two weeks at the beginning and two weeks at the end of the semester) which included preservice teachers' frequent visits to mentors' classrooms as well as visits by university faculty and staff. Preservice teachers had the opportunity to plan and teach a one-week unit lesson in his/her's mentor's classroom. Additionally, participating PRIME mentors were required to attend a half-day PRIME orientation and four evening work sessions that focused on professional development and reforming the teaching of mathematics.

During its three years, Project PRIME has involved approximately 50 secondary mathematics teachers, 60 preservice teachers, and 15 university faculty and staff. Although the 2000 – 2001 academic year was the conclusion of Project PRIME as a funded Eisenhower Project, PRIME has continued as Program PRIME, which means the purpose and structure of Project PRIME has continued to be an integral part of secondary mathematics teacher education program at the University of Georgia. The nine teachers who volunteered for this study mentored preservice teachers who were supervised by three of the authors. Four of the participants were charter members and three-year participants in PRIME, three teachers had participated in PRIME for two years, and the 1999 – 2000 academic year marked the initial participation for the remaining two teachers.

Procedure

Each teacher participated in three different audio-recorded interviews. The first two interviews were semi-structured and consisted of seven questions framed by a discussion of successful mathematics lessons. The questions were indirectly attempting to motivate conversation regarding the participants' practices of good mathematics teaching rather than reveal their hypothetical beliefs about good mathematics teaching. Therefore, the question What constitutes good mathematics teaching? was never asked of the participants. The questions from the first interview focused on the teachers' practices, e.g., Describe what you consider to be one of your most successful lessons. Why is it such a good lesson? The second interview centered on the teachers' views of their student teachers' practices, e.g., What was the best lesson you have observed by your student teacher? Describe. Why was it such a good lesson? What would you suggest to the student teacher that could make it an even better lesson? The questions were based on Schön's (1983) theory about how practitioners reflect in and on their own practice. Before the third interview participants were provided transcripts of the first two interviews and were asked to highlight those points they deemed important. This third interview focused on what the teachers highlighted and asked them to elaborate those points. The third interview was also transcribed.

The three transcribed interviews resulted in approximately 135 pages of data. Using qualitative methods of coding interview data as "a progressive process of sorting and defining and defining and sorting" (Glesne, 1999, p. 135) an initial coding scheme of twelve codes was developed. A line-by-line analysis identified three main categories of conversation in which

teachers engaged. They were: characteristics of good teaching, characteristics that separate an experienced teacher from a novice, and how teaching knowledge was developed. Since this study was attempting to determine what the secondary mathematics teachers perceived as good mathematics teaching, the categories of characteristics of good teaching were further analyzed and additional codes developed, resulting in a coding scheme of eighteen codes. We again performed a line-by-line analysis of the data and produced a frequency table according to these eighteen codes. A final synthesis of the "sorting and defining and defining and sorting" revealed "data clumps" (Glesne, 1999, p. 135) that led us to infer various aspects of good teaching, according to the teachers' statements.

In addition to the collection of interview data the researchers met with a working group of seven PRIME teachers during a one-day summer 2001 work session to discuss the data analysis and initial findings of the study and to receive input from these teachers as to what they thought constituted good mathematics teaching; only one of the teachers in this summer work session was a participant in the study. The work session was audio-recorded and analyzed in order to provide additional support to the researchers' conclusions. Both the summer work session and the third interview offered face validity to the research study, as "analytical categories and emerging conclusions were continually recycled back through the respondents" (Lather, 1986, p. 74).

Teachers' Perspectives on Good Teaching

As teachers discussed their best lesson in the first interview, the attributes of their student teachers in the second interview, and summary points in the third "review" interview, they painted a picture of what good teaching meant to them. Although the brush strokes varied from teacher to teacher, the composition of the individual paintings was the same. All nine teachers emphasized the importance of prerequisite teacher knowledge, promoting mathematical understanding, engaging students, and effectively managing the classroom environment.

The following sections explain the nature of these four characteristics of good teaching. Using quotes from interview transcripts, we have tried to capture the wisdom of the group and at the same time delineate the similarities and the differences in teachers' perspectives.

Good Teaching Requires Prerequisite Knowledge

Our interviewees reported that mathematics teachers need to have an extensive knowledge of both mathematics and their students in order to teach well. They need a knowledge of

mathematics in order to teach for understanding, sequence lessons, make transitions between topics, understand student questions, provide good examples, and maintain the necessary confidence in front of a class of students. Seven of the nine teachers explained how mathematical knowledge supported good teaching. Yvonne focused on the importance of a comprehensive knowledge of mathematics.

I think that a lot of mathematics comes from knowing the upper level math. I can tell my class and I can exude a love of mathematics, even when I am talking about the reflexive property, because I know how it fits into the bigger whole. So I think there is no way we could train people to be good mathematics teachers in high school, if we don't expand their ability to do the math and to love it. (1, Yvonne)

Alice argued for mathematical knowledge to help a teacher develop mathematical confidence.

...that whatever they throw at you, even if you can't answer it, you are going to have the confidence to explore it with them or to say, you know, I don't know the answer to that right now but we are going to find out. That's not easy to do if you're not feeling really good about what you know. (3, Alice)

Some teachers alluded to not only knowing more mathematics but knowing it in a way that supported the teaching strategies that they wanted to use. For example, Melanie explained, "I would like to see more ... how to come up with different examples, how to look at things from different angles." (1, Melanie) Anna thought it was important to know enough mathematics to make connections between mathematical ideas and create useful transitions between topics. She argued that teachers need to know

just little things like how to transition from one section [of text book] into a new section... to know why [the sections] are in that order and to think about why they are in that order...I will rearrange the order that I teach it or sometimes I will even make the decision to go ahead and teach it [the existing order]. (1, Anna).

The mathematical knowledge that the teachers valued consisted of more than being able to work all the problems. They were calling for knowledge of the structure of mathematics. They were arguing that mathematical knowledge provides confidence and a freedom to move away from a textbook and a rigid curriculum. A good knowledge of mathematics allowed the teacher to vary the lesson and make it more interesting.

Teachers also emphasized the need to know your students. They tended to emphasize personal knowledge of the student or knowledge of the students' skill levels in contrast to knowing how a student was learning or constructing mathematical ideas. Nick warned, "... a student teacher just needs to keep in mind that these are ninth grade students taking it [Algebra I] for the first time and stop and think what they know, don't take it for granted." (1, Nick) At another point in the interview, he explained that one of the most important assets in a student teacher is the ability to "maintain a professional relationship with your kids and at the same time understand the difficulties and the problems they are having as a ninth grader." Melanie advised that student teachers should "look back through the book to see what we have done previous to them attending or coming to teach so that they will know where to start with the students (3, Melanie). Again, referring to a student teacher, Holly explained, "She's gotten better at learning what the students need to know, prerequisite skills, making sure they are prepared to do that lesson." (2, Holly) Teachers wanted their student teachers to know what mathematics had been studied, leaving the impression that if it had been taught, you could assume that most students knew it. They also wanted their student teachers to be sensitive to the needs of high school students, and to be attentive to personal problems that students have outside of the mathematics classroom.

Good Teaching Promotes Mathematical Understanding

Teachers in our study frequently explained that their goal was to teach so that their students understood the mathematics. They wanted their students to be able to use their mathematics outside of the classroom as well as in the study of other mathematics. As teachers described good lessons, we inferred different types of mathematical understanding that we coded as procedural, conceptual, or related to the nature of mathematics. Two teachers explained that they wanted their students to understand the versatility of mathematics. Only a few comments referred to the importance of procedural learning. Nick praised a student teacher for going "step-by-step, so the kids had a system they could use to get the correct answer" (2, Nick), and Melanie was pleased that the student teacher was "making it more concise for the students so that they can understand it better." (2, Melanie). Some of the teachers' statements were ambiguous and could have been interpreted as wanting students to understand either a procedure or a concept. For example, a teacher reported that her student teacher walked around the room and was able "to get a better idea of whether or not students really understood" (2, Yvonne), but she

did not specify what was to be understood. It is important to note that more than half of the coded statements related to understanding referred explicitly to a conceptual understanding. Upon a reflection of her learning experiences, one of the mentors provided a description that gave us a working definition of conceptual understanding.

I really didn't start enjoying math until I really started understanding it because just to memorize things and do things because the teachers tell you, to me is, you know, that's not good enough. So it's through his work that he (previous mentor) did, that I come to teach the same way. I am very big on kids exploring a topic or thinking about it before I just start throwing definitions at them. (I, Elizabeth)

All of the teachers interviewed were interested in student understanding that moved beyond the instrumental understanding involved with using a formula or explaining a mathematical procedure. The teaching strategies that they recommended, their described uses of technology, and the reported mathematical activities were supported by the teachers' arguments that these practices would help students to understand the mathematics.

As teachers talked about good teaching, several strategies or pedagogical goals were repeated and emphasized. In referring to their own teaching or discussing strategies that student teachers needed to develop, our interviewees highlighted: connecting mathematics, visualizing mathematics, and assessing students' understanding. They also emphasized the need to refrain from just telling students what you want them to know because it was not an effective way to help students understand.

Connecting mathematics. Seven of the nine teachers made explicit references to connecting mathematics. In general, the reason to make connections was to enhance student understanding but the kind of connections advocated varied. Some said it was important to make mathematical connections and smooth transitions between mathematical topics. Peggy explained, "You can do it graphically, you can model it algebraically, there are just so many different things that you can do with it to kind of help the kids understand why ... and where it is going." (I, Peggy). Brian explained what he was hoping to instill in student teachers,

I would like for them to show... how everything fits together, so that the students do not just see a formula, and learn how to substitute numbers, so that they see the relationship between the geometrical drawing and the application of it. I guess the most

important thing I would want the student teacher to stress is, where the idea comes from, and why we do use it, and what areas we would want to use it in. (2, Brian)

Other teachers valued connecting mathematics to the world of the students and making applications of the mathematics specific. Teachers varied in how they introduced the connection, but many wanted the connection to be a concrete example or specific application.

"One of the reasons it [lesson on slope] was so successful is because there are so many examples, the students could see, feel, and touch, and it really brings it home to the students. (1, Nick).

"I was trying to bring in student activities ... that they were doing outside of the classroom. " (1, Holly).

"Once I saw the forms and realized that I could tie [mathematics] into literally everything in creation, yes, that is very powerful for me." (2, Alice)

"I could do this [science] lab in a math class, because what they were studying was theoretically.. a linear relationship (2, Anna)

[My student teachers have] have been good in coming up with in-class lab-type, hands-on experiences for the students. It helps the students; it gives it more of a connection." (2, Melanie)

A few teachers emphasized placing the mathematics in a context and enthusiastically discussing all that the study of mathematics offers.

Don't just show them problems and when you see this problem, this is what you do, but try to help them understand that mathematics is much more than just balancing your checkbook. Talk about it as a science of patterns, and ideas, and encourage your students by being enthusiastic yourself. (1, Alice).

Melanie explained, "I just feel that through teaching you need to make math more real to the students. It can be a dry subject and it needs to be more real." (3, Melanie) Teachers

believed that if they made connections, students would understand the mathematics or, at least, they would be more motivated to try to understand the mathematics. The responsibility for making the connections belonged to the teacher; the student's responsibility was to understand.

Visualizing mathematics. Four of the nine teachers made specific references to helping student visualize mathematics by using computers or calculators, drawings, or concrete materials.

The real world demonstration of using the houses and the telephone poles. . . makes it more real to them. Okay, this is how it can be used, and I think when they can picture the math or the Geometry in this case, I really think it helps them to retain it more. (1, Melanie).

Elizabeth argued for the power of using visual representations to increase understanding about the unit circle and trigonometric functions.

Letting them see that visual and it actually moving ...I could say 'see at ninety we are here on the graph.' After I was done with that—this is a pre calculus class—a student said to me that 'I have never understood that until this moment.' And he said 'Wow, that is so cool!' Just hearing one student say that made me realize that I think that was successful. Usually hands-on or calculator is going to be where I feel I am most successful. (1, Elizabeth)

Alice noted that a drawing can resolve a great deal of confusion and serve as a tool for understanding.

The first two years that I taught Trigonometry, I used that crazy rule, and neither the kids nor I could ever keep straight what was B, and what was A, until we started drawing them. It was kind of an outgrowth of why do we want to memorize this silly rule, why don't we just look at a diagram and understand why it is acting the way it is, and have a deeper understanding than just, here is a rule. (1, Alice)

Teachers who advocated a visual approach emphasized it throughout the interviews. Some teachers did not specifically talk about a visual approach but discussed using hands-on materials and laboratory activities that suggest a visual and tactile approach. Occasionally, the strategy appeared to be motivated by a need to keep the students' attention, but usually the teachers advocated a visual presentation as an effective means of helping student "see" the mathematics.

Assessing students' understanding. Six of the nine teachers mentioned the need to assess student understanding. The need was made explicit, but the nature of the assessment was not deep. Teachers talked about student teachers needing to know what had been taught with the implicit assumption that what was taught became part of the learner's knowledge. There were also suggestions about "looking into the eyes of the students, and see what they need." (1, Anna). Yvonne praised her student teacher by saying, "She actually got away from the comfort zone, out into the classroom...was able to get a better idea of whether or not students really understood" (2, Yvonne). Nick advised,

Make sure that, to check on understanding...whenever you are teaching a lesson or teaching a new concept.. don't talk too long before you stop and you say, OK, let's see if you understand what I have done so far and try.. it's important to stop at intervals and check. (2, Nick)

Teachers reported a need for frequent, quick evaluation that would inform the instructor whether or not to proceed. The quick assessment was important to inform the instruction, but teachers did not include substantial assessment as part of the good lessons they were describing. For example, the interviews did not uncover examples of probing students for the limits of their understanding or challenging students to explain where they were having difficulties.

Refrain from telling. Eight of the nine teachers discussed good lessons as moving away from the teacher providing information that the students had to memorize or mimic. Brian praised a lesson taught by his student teacher, saying "The best thing about it was just their [high school students'] participation, they were demonstrating what they knew, what they brought into class, and being able to make their own conclusions." (1, Brian)

Yvonne explained why mathematics required a non-telling approach.

...mathematics is different because it is just not giving content, but it is continually assessing more understanding... so it is not just giving a lecture...it is developing something, modeling something, and showing something...Mathematics involves a lot of thinking, so it is not just listening, writing down notes. (1, Yvonne)

Teachers realized the inadequacy of telling students; they realized that it did not lead to their goal of student understanding. They argued that just knowing what the teacher tells you does not prepare you to solve real problems or use mathematics outside the classroom.

The fact that I wasn't just telling them a definition and writing it on the board ...those kids will memorize the definition and they would be able to spit it back to me, no doubt in my mind, but what do they really understand? (1, Elizabeth)

They can watch me do a problem all day on the board but they are not going to learn it until they try to figure it out themselves and I guess that's a big philosophy. I think the student does need to know how to do. Math not only teaches you basic formulas for doing things in life but also it helps in problem solving, logic, figuring out logic. (3, Melanie)

It is easy enough to claim that telling is not an effective strategy, but the more complicated issue is to decide what should replace telling. Teachers wanted students to discover properties and ideas, and they were dedicated to using activities, employing technologies, and designing lessons that led to a specific conclusion. Teachers expected students to discover a piece of mathematics that the teacher thought was important. The teachers worked hard to guide students and, on occasion, advocated directly leading students to the desired conclusion. Teachers maintained the major responsibility. Teachers tried to balance what needed to be told and what could be discovered, but the goal was clearly for the students to learn what the teacher had planned.

We were developing the product rule and the quotient rule... it was a discovery lesson where we could use techniques that they knew...we discovered the chain rule and some other ways...I like the 'oh' at the end of the lesson, but there are other lessons that are direct teaching, that also give you that 'oh' at the end. (1, Yvonne)

I like them [students] to, when possible, have hands on experience, discover things, do some of the preliminaries on their own so that they get that ah ha experience. I don't think that's probably very different than you would have heard say 10 years ago. I don't subscribe to the fact that you can just let them discover and that you can make the assumption that they are going to make all of these links that you've made. After all, you

designed the lesson. Of course you made the links. ... I guess I am saying, I love for them to work hands-on and discover. (3 Alice)

... not telling them [students] ahead of time what the relationship would be... if you tie it all together—get them to tie it together, not telling them necessarily how it all fits together, but making sure before they leave the room they've seen how it ties together. (2, Anna)

Teachers seemed to feel a pressure to be sure that the students understood "the concept" before they left the class period. If the "discovery" did not work, it was necessary to do more guiding and move towards telling.

Good Teaching Engages and Motivates Students

The importance of engaging students was discussed by eight of the nine teachers. The idea of engaging students is very compatible with the teachers' emphasis on understanding and with their belief that "just telling" is not an adequate means to promote understanding.

One of the most frequently discussed ways to engage students was to have a variety of types of lessons. Teachers mentioned using technology, having students write, doing group work, hands-on activities such as measuring distances, applying mathematics, and laboratory activities. Sandra explained the importance of varying lessons.

I know that every day you can't come in here and be an entertainer, I don't mean that. But, come with new activities or new ways to do things so that kids don't just, OK, open the book. ... I've had really good experiences with that. (2, Sandra)

Anna praised her student teacher for variety and explained what was necessary for creating variety.

My student teacher is very knowledgeable of mathematics. She's not having to struggle at all with any of the content, so I think that frees her up to think about different ways that she can teach the material so that the students won't get in a rut...[she] gives them some freedom to do different things and to think. (2, Anna)

One of the popular ways to vary instruction was to use groups of students. Teachers argued for group work as a way for students (1) to help each other, (2) to share ideas, (3) to do problem solving and (4) compile and validate measurement data. None of the teachers talked about a

formal structure for the group work or creating cooperative working groups. Although teachers argued that "teacher telling" was not useful, some of the value of group work was reported as an opportunity for students to tell other students how to work problems. Elizabeth praised her student teacher's use of groups and especially group assessment.

I liked how they worked in groups and she'd come around and have one person answer the question that she threw at them. ... they were all involved and they all wanted to make sure that everybody in the group knew, so I thought that was great. (3, Elizabeth).

Four teachers specifically referred to students physically moving around the room in order to engage them. Movement was also implied in the activities of other teachers who advocated groups or working on an activity. Teachers commented.

I thought that [the lesson] was a good activity to do because it got them [students] up and moving.. (1, Anna)

There needs to be more, really involvement and students getting out of their desks and movement and different types of activities.(2, Yvonne)

They are not simply looking at a process written down on a board, and repeating it, or regurgitating information that someone has given them, or filling in a formula. They feel like they are part of the problem because they have gotten physically involved with it. (1, Alice)

Sometimes the physical movement was as mundane as going to the board to explain a problem or moving to work with a partner, but physical movement was a substantial part of engaging students.

Teachers also recommended engaging students by meeting them at their mathematical level. They praised student teachers who could "come down to their [students'] level", when they are "used to thinking way up there." (2, Elizabeth). Although teachers wanted to reach high school students at the level where they were, two teachers cautioned that we need to continue to challenge students even when it is uncomfortable. They recognized the problem of simplifying a concept and still holding high expectations for their students.

People have a tendency to want to stay in their comfort zone and if you stay in your comfort zone you don't really grow. We've gotten away from challenging them just a little bit, not too much because it is detrimental to them...Nobody wants to make anybody else uncomfortable and a lot of people don't want to get out of their comfort zone so we don't push them in the directions as much as we should. (3, Peggy)

... if they struggle with this, they [students] tend to make it their own and they are going to be proud of it in the end. And maybe prouder of making a B in geometry than they are of an A that they made in another class because they had to work and it's really theirs and they created their own knowledge. (2, Anna)

These last two comments on engaging students through challenging them were not typical, but they do exemplify the challenge teachers face when trying to (1) engage students at their existing knowledge level and (2) inspire them to move to a less comfortable, but more advanced zone.

Part of engaging students involved motivating students. It seemed that the teachers valued certain teaching strategies because they helped students understand which, in turn, motivated students. There were also strategies there were designed primarily to motivate with the hope that the engaged student would then understand. One teacher claimed, "The number one job of teaching is motivating them to want to do it." (1, Elizabeth). In a subsequent interview, she explained,

So that's my basic philosophy—if you let them see why and see the enjoyment in it and how it's neat, how math fits together so well, it hopefully will motivate them to want to learn more. .. my whole philosophy is concentrate on exploring it on your own somewhat, so it means something to you. (3, Elizabeth)

Another teacher elaborated on making mathematics fun saying, "Let them know, that even if math is not your favorite subject, it can be fun and you can learn it." (1, Anna)

Good Teaching Requires Effective Management

As teachers described good teaching, they usually mentioned the necessity of good management to achieve other goals. Three teachers emphasized management as most important for student teachers.

Well, I think number one is classroom management....how to handle themselves with a group of thirty freshmen ...taking you in different directions at the same; how to

handle stress. ... If you can't handle the stress and you can't get their attention and you can't manage the kids, it doesn't matter what you use, you're not going to be successful. (1, Nick)

I said to cope with everyday situations. ... Planning the lesson is good but they really need to see and experience all of those things that you have to do everyday, attendance and letting them go to the bathroom, assemblies, and how all of these things affect the day. That's just a practical trade need. (3, Brian)

Classroom management. I think is the most difficult thing that any student teacher had to deal with. Because if you just had an ideal class the things you get to try are great. But getting them all to be involved in it is another whole story—working with just ways to get that class involved, ways to keep them the whole time. (2, Elizabeth).

For most of the teachers, "management" covered a host of skills and ideas to keep the lesson moving and the students under control.

Managing your time, managing your students, dealing with all different things, record keeping, devotion to the kids, just getting the lesson plans together, be sure that you are staying where you should be, just learning how to pace yourself so that you don't drive the kids too hard, but you do push them so they do stretch themselves, you don't get too far behind because one of the most difficult things I think for new teachers is realizing that you don't have all week to cover one section. (1, Peggy)

You could hear the frustration in many of the comments. There is so much to consider simultaneously. Teachers argued that much of the management is learned through experience, but they also praised student teachers who seemed to be managing well. Some reflected on their own difficulties saying the first year was the most difficult. Although a few teachers spoke of wanting the students to walk away with a "bag of tricks" and one teacher wished more could be addressed before student teaching, all teachers commented that you learn through experience in a classroom.

Another aspect of having effective management skills was the notion of flexibility which was repeatedly mentioned by the teachers. As Yvonne put it, teachers need "To roll with the

punches, be flexible. Flexibility is a sign of good mental health...shake off one day if it is a bad one, and look to the next day." Anna thought of flexibility in the following way.

I think all of that is contributing to her [student teacher] being a better teacher because she is realizing that it's not just her plan, that you have to kind of roll with the flow and I think you have to be comfortable doing that...I think you have to meet the students where they are. (2, Anna)

Alice seemed to be referring to teachers' flexibility in their pedagogical content knowledge when she said, "I think it would be very important for them to be prepared, to have thought of as many ways as they can to describe whatever it is they are trying to do." Flexibility was a theme that permeated teacher's descriptions of their lessons and their comments about lessons of student teachers. Flexibility was a management technique, and the purpose was to keep students engaged and motivated. They wanted to find a hook that would meet students where they were. Rather than trying to understand student thinking or how the students were building concepts, teachers recommended taking a variety of approaches with the assumption that there would be at least one approach that would reach each learner.

...not all learners learn the same way. That probably is the biggest thing I've learned as a teacher is what worked for me doesn't necessarily work for another child, realizing that there's different learners and just because one method doesn't work doesn't mean you can't try another method and that may be the thing that turns the light on for them. (3, Elizabeth)

Elizabeth recognizes student differences and advocates multiple approaches, but never discusses how one might thoroughly assess student understanding.

Hearing the Echoes

Although teachers seldom use the words of researchers or scholars of decades past, to what extent did these nine teachers reflect the ideas expressed by those individuals? Without question, we hear the teachers emphasize the need to know mathematics, the foundation on which their teaching rests. Yvonne feels that "a lot of mathematics comes from knowing the upper level math" suggesting that teachers need a deep and thorough understanding of the mathematics they teach. The importance of knowing mathematics is based on many considerations, not the least of which is the confidence it gives the teacher. As Alice put it, "you are going to have the confidence to explore it with them or to say, you know, I don't know the answer to that right

now but we are going to find out." Anna suggests a slightly different orientation in that a thorough knowledge of mathematics allows the teacher to make better transitions "from one section [of a textbook] into a new section...to know why [the sections] are in that order and to think about why they are in that order." This is in keeping with Polya's (1965) commandment "Know your subject," and with the very premise of the various NCTM Standards.

There are other kinds of knowledge that teachers need to have as well, for example, knowledge about their students as individuals and as mathematical beings. Repeatedly, the teachers emphasized the need to know their students to be able to identify with them as emerging adults. But they also emphasized understanding students' understanding of the mathematics to be learned. Recall Nick's admonition that student teachers need "to keep in mind that these are ninth grade students taking [Algebra I] for the first time and stop and think what they know" and that it is "important to stop at intervals and check." Despite the teachers' emphasis on understanding students' thinking there remains the question of whether they are assessing students' understanding as a means of shaping their instruction or whether their assessment consists primarily of determining the extent to which students are "with the teacher" as the class follows a rather predetermined curriculum. The interviews seem to suggest that the teachers considered it important to assess so as to appropriately pace instruction rather than shape instruction. This is somewhat different from Schifter's (1998) constructivist-based notion that the mathematics to which teachers should be attending is not that of a predetermined curriculum but rather the mathematics in the students' mind. Schifter emphasizes the need for teachers to base their instruction, and not just the pace of instruction, on students' mathematics. There appears to be some variance between how teachers define good teaching in terms of assessing students' understanding as they move along a predetermined curriculum and the constructivist notion that that assessment is what determines what gets taught next. Schifter's constructivist perspective does not seem to capture what the teachers have in mind. Sanchez (2001) and Lee (2002) both found that teachers' assessment practices were primarily for determining grades or the pace of instruction and not for the purpose of shaping instruction or accommodating students' thinking.

Davis and Hersh (1981) maintain the importance of students seeing and experiencing mathematics as a pluralistic subject and something other than a formalized science. A certain pluralism pervaded the teachers' comments albeit perhaps not in the same sense Davis and Hersh

intended. Melanie pointed out that it is good to see "things from different angles." Peggy's perspective that you can represent concepts graphically, algebraically along with other ways reflected a certain pluralism. Perhaps Alice echoes Davis and Hersh best when she offered that you can "Talk about it [mathematics] as a science of patterns, and ideas" and later added "Once I saw the forms and realized that I could tie [mathematics] into literally everything in creation, yes, that is very powerful for me". Alice's inclination to provide students with hands-on experiences, opportunities to discover mathematics and to have "ah ha experiences" further suggests a pluralistic approach to the teaching of mathematics that, at least to some extent, reflects Davis and Hersh's perspective. Further evidence of a pluralistic approach to the teaching of mathematics is reflected in the teachers' emphasis on visualizing mathematics as discussed in a previous section.

Polya's (1965) mandate that you "Suggest it, do not force it down their throats," was frequently reflected in the teachers' remarks as they saw the teaching of mathematics as not just a matter of telling students. The notion that students have to figure things out for themselves captures both Polya and the teachers' perspectives. Yvonne's explanation illustrates this perspective. "mathematics is different because it is just not giving content, but it is continually assessing more understanding...so it is not just giving a lecture...it is developing something, modeling something, and showing something. Mathematics involves a lot of thinking, so it is not just listening, writing down notes." Anna adds, "not telling them [students] ahead of time what the relationship be...get them to tie it together, not telling them necessarily how it all fits together."

Polya's (1965) emphasis on teaching students guessing (commandments 6 and 9) are reflected in Elizabeth's call for letting kids "explore a topic or thinking about it before we start throwing definitions at them." The tension here is that Elizabeth seems to emphasize the importance of letting kids explore, which could be interpreted as guessing, before they get to the formalized mathematics with definitions. That is, it is kind of a sweetener to make the mathematics more palatable. Polya, on the other hand, seems to be suggesting that teaching students guessing (and proving) is, in many ways, the main course—not some sort of additive, whereas Polya's emphasis is on guessing as an end in itself. Fundamentally, there is a big difference between these two perspectives.

A various number of Rosenshine and Furst's (1991) variables appeared during the interviews. Prominent among them, at no surprise, is the importance of a teacher being enthusiastic (recall also Polya's (1965) first commandment), a consistent theme mentioned by the teachers. Melanie who was pleased that her student teacher was "making it more concise for the students so that they can understand better" alluded to a certain emphasis on clarity. If the notion of flexibility is similar to Rosenshine and Furst's notion of variability, then this too was reflected in what the teachers considered good teaching to be as they repeatedly mentioned the need to be flexible in not only dealing with students but in using a variety of instructional methods to promote understanding. Rosenshine and Furst's analysis of variability is probably more aligned with this latter notion of flexibility.

With respect to effective management, the teachers did not raise the issue of time allocation as did Good, Grouws, and Ebmeier (1983) but underlying many of their comments was the notion of efficiency—an emphasis raised by McKinney (1986) and Leinhardt (1988) in their comparison of expert and novice teachers. Efficiency was raised by one teacher who thought "one of the most difficult things I think for new teachers is realizing that you don't have all week to cover one section." But, for the most part, teachers thought of effective management as the ability to cope with a variety of changing circumstances over a period of time. Not surprisingly, classroom management in terms of the myriad of tasks teachers have to deal with was at the center of most of their concern about effective managing of the classroom. It was in this context that the teachers' notion of flexibility arose.

Overall, the teachers reflected a conceptual orientation toward teaching. It also seems fair to say that their comments reflected an essentially teacher-centered classroom although not one dominated by lecture. To some extent this reflects what Wilson and Goldenberg (1998) found in that Mr. Burt did reform his teaching in the sense of emphasizing conceptual outcomes but not in the sense of changing his teaching style from a teacher-centered approach to a student-centered approach. One got the feeling that the nine teachers feel comfortable with a teacher-centered approach so long as their instruction uses a variety of instructional styles and they find different ways of reaching out to students. To say it another way, one might conclude that they saw their teaching as problematic but they did not necessarily see the mathematics that they were teaching as problematic—that had been determined by county or state objectives or textbooks. Neither did one get the feeling that they wanted to avoid frustrating the students at all costs as suggested

by Gellert (2000). Although Gellert was working with an entirely different population (preservice elementary teachers), it is not uncommon for teachers to espouse a caring attitude toward students, sometimes at the expense of the mathematics involved (see Cooney, Wilson, Albright, & Chauvot, 1998). This orientation did not seem to be the case with our teachers. Nevertheless, although the teachers embraced part of the NCTM reform-based notions about teaching and learning, that acceptance was conditional—a perspective not unlike that found by Eggleton (1995) and Chauvot (2000) in their work with preservice secondary teachers.

Concluding Thoughts

Dewey thinks of education as a reconstructive process that is grounded in reflective thinking and allows the individual being educated to make more informed choices. These are lofty goals, indeed. Did the teachers convey the essence of Dewey's philosophy? Yes, in the sense that the teachers wanted their students to understand mathematics as it related to other aspects of their lives. As Yvonne put it, "Don't just show them problems and when you see this problem, this is what you do, but try to help them understand that mathematics is much more than just balancing your checkbook." But we did not uncover much of an emphasis on promoting reflective thinking or on students learning how to monitor their own learning. Thus, it seems that the teachers reflected those notions of good teaching that have viability in a teacher-centered classroom. This does not negate their attentiveness to their students nor to helping them understand mathematics from a variety of perspectives. Pluralism abounded throughout the interviews.

It should be kept in mind that these were experienced teachers who take considerable pride in their profession of teaching mathematics. Based on their participation in Project PRIME it would be reasonable to conclude that they were well established, well respected teachers by both students and parents alike. At the very least, they were initiators and leaders in their own school districts. So perhaps it should not be surprising that their notions of good teaching echoed that of many scholars and researchers who have written about the teaching and learning of mathematics. Although the teachers may not hold to Dewey's notion of good teaching in its broadest sense, their notions are grounded in a substantial body of literature.

The teachers' perspectives also seemed to reflect much about the NCTM Standards. The notion of solid mathematics, connecting and visualizing mathematics, and engaging students in doing mathematics are all precepts that pervade the Standards. Further, the teachers use of

technology, specifically connected to visualizing mathematics, is another indication that the teachers are "living" the Standards. The following statement in the Standards

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well (NCTM, 2000, p. 16) was illustrated many times by the teachers albeit it is not clear that they fully embraced some of the constructivist leanings in the Standards. Whereas the Standards present effective teaching in terms of the teacher basing instruction on students' thinking, this orientation is probably not consistent with how the teachers' interpret assessing students' understanding. The teachers seem to maintain that it is important to understand what the students understand but more from the perspective of knowing whether they are going too fast or whether they need to reteach a topic. There was no evidence in the interviews that teachers spent time trying to understand how students were constructing ideas or using assessment to probe knowledge. Although teachers valued conceptual learning, they spoke as if a concept was known or not yet known showing little insight into the gradual development of complex concepts. They were interested in whether to move on or the back up in the teacher-directed lesson. Students had minimal influence on the content or the direction of the lesson.

A caveat seems appropriate. This study was based on interviews, teachers' perceptions, as they responded to the questions posed. It is not based on observations on their classes nor on the ways that they actually teach mathematics. Thus one cannot conclude that what they believe about good mathematics teaching parallels the way that they teach mathematics. Too, we never asked the teachers, directly, to define good mathematics teaching. Nevertheless, we can reasonably conclude that we captured their ideal of what constitutes good teaching and how it should be taught.

From this perspective, we can take comfort in the fact that one can find considerable residue from the teachers' espoused beliefs in the writings of those who have spent a considerable amount of time thinking and reflecting on the act of teaching. Teachers' wisdom and the wisdom of the scholars may not be the same, but there is considerable overlap. There is also a significant point of departure between the teachers and the interpretivist scholars. Schifter (1998) is calling for teachers that see mathematics in terms of their students' constructions of mathematical ideas, and this approach to good teaching was not part of the thinking of teachers in this study. This poses a significant challenge for teacher educators or reformers who advocate

a more student centered approach to teaching. The teachers in this study were conscientious, reflective, and their descriptions of good teaching were in line with many NCTM principles, but their thinking about good teaching was tied to teacher-directed lessons. Are the experience-based ideas of the teachers compatible with the shift to student-centered teaching? Can conscientious teachers shift their responsibilities from controlling a well-managed lesson to exploring student understanding? Are teacher educators and teachers focused on the same goals? There is a danger that when we discuss good teaching, teachers and teacher educators are picturing the products of good teaching in different ways. In order to improve the conversation, teacher educators need to listen to how teachers are thinking about good teaching.

References

- Brownell, W. (1956). Meaning and skill—Maintaining the balance. The Arithmetic Teacher, 15, 129-136.
- Clark, J. & Fehr, H. (1953). Learning theory and the improvement of instruction—A balanced program. In H. Fehr (Ed.) The learning of mathematics—Its theory and practice (pp. 335-349). Washington, D.C.: The National Council of Teachers of Mathematics.
- Chauvot, J. (2000) Conceptualizing mathematics teacher development in the context of reform. Unpublished doctoral dissertation. University of Georgia. Athens, GA.
- Cooney, T., Wilson, P., Albright, M. & Chauvot, J. (1998). Conceptualizing the professional development of secondary preservice mathematics teachers. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Davis, P. & Hersh, R. (1981). The mathematical experience. Boston, MA: Birkhäuser.
- Dewey, J. (1916). Democracy and education. New York: The Free Press.
- Eggleton, P. (1995) The evolving mathematical philosophy of a preservice mathematics teacher. Unpublished doctoral dissertation. University of Georgia. Athens, GA.
- Gellert, U. (2000) Mathematics in safe place: Prospective elementary teachers' views of mathematics education. Journal of Mathematics Teacher Education, 3, 251-270.
- Glesne, C. (1999). Becoming qualitative researchers: An introduction, (2nd ed.). New York: Longman.
- Good, T., Grouws, D. & Ebmeier, H. (1983). Active mathematics teaching. New York: Longman.

- Ice, N. (2002). Algebra I teachers' use of open-ended assessment items. Unpublished doctoral dissertation. University of Georgia. Athens, GA.
- Lather, P. (1986). Issues of validity in openly ideological research: Between a rock and a soft place. Interchange, 17 (4), 63 – 84.
- Leinhardt, G. (1988). Expertise in instructional lessons: An example from fractions. In D. Grouws, T. Cooney, & D. Jones (Eds.) Perspectives on research on effective mathematics teaching (pp. 47-66). Reston, VA: National Council of Teachers of Mathematics.
- McKinney, K. (1986). How the experts teach math. In Research in brief. Washington, DC: U.S. Department of Education. Office of Educational Research and Improvement.
- National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.
- National Council of Teachers of Mathematics (1991). The professional standards for teaching mathematics. Reston, VA: Author.
- National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, VA: Author.
- Polya, G. (1965). Mathematical discovery: On understanding, learning, and teaching problem solving (Vol. II). New York: John Wiley and Sons.
- Rosenshine, B. & Furst, N. (1971). Research in teacher performance criteria. In B.O. Smith (Ed.) Symposium on research in teacher education. Englewood Cliffs, N.J.: Prentice Hall.
- Sanchez (2001) Conceptualizing mathematics teachers' use of open-ended assessment items. Unpublished doctoral dissertation. University of Georgia. Athens, GA.
- Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. Journal of Mathematics Teacher Education, 1, 55-87.
- Schön, D. (1983). The reflective practitioner: How professionals think in action. New York: Basic Books.
- Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. Journal of Mathematics Teacher Education, 4, 3-28.
- Wilson, M. & Goldenberg, M. (1998). Some conceptions are difficult to change: One middle school mathematics teacher's struggle. Journal of Mathematics Teacher Education, 2, 269-293.